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Two-dimensional traffic flow problems with faulty traffic lights

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A cellular automaton model is presented to simulate a traffic jam in a two-dimensional traffic flow with faulty traffic lights. The model is studied by computer simulations. Results show that at low car densities, faulty traffic lights speed up the overall traffic. However, the critical car density at which a serious traffic jam occurs decreases as the concentration of faulty traffic lights increases.

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I. INTRODUCTION

Recently, there has been much interest in the study of traffic flow problems within the context of cellular automata (CA). This approach has the advantages of being simple and computationally less intensive, and can be easily modified to study different effects. It is analogous to the application of the cellular automata to hydrodynamic problems. Biham, Middleton, and Levine [1] (henceforth referred to as BML) introduced a simple two-dimensional (2D) CA model with traffic lights and studied the average velocity of cars as a function of their concentration. In the original model, cars moving from west to east will attempt a move in odd time steps, say, and cars heading from south to north will attempt a move in even time steps. It was found that the average velocity in the long time limit vanishes when the concentration of cars is higher than a critical value. The basic CA rules have been generalized [2–6], for example, to include the effects of two-level crossings (overpasses) and accidents on the road. Traffic flow problems in one direction have also been studied using one-dimensional cellular automata (CA) models [4,7–10].

In this work, we study the effects of faulty traffic lights in the two-dimensional CA model introduced by BML. In the BML model, cars are placed in a $N \times N$ lattice, with equal numbers of northbound and eastbound cars. North-bound (eastbound) cars will only attempt a move in odd (even) time steps, so as to regulate the traffic flow to prevent two cars from moving into the same site simultaneously. Thus, the even and odd time steps represent the traffic lights in the model. When a car attempts a move, it will move forward a step unless the site in front of it is blocked by another car. In that case, it will not move, even if the blocking car leaves on the same time step. The effects of faulty traffic lights can either lead to a higher average speed of cars or a slower traffic, depending on the car density. For low car densities, the chance of cars blocking each other's way is low. In this case, turning off the traffic light may even lead to faster traffic, as cars do not need to stop at traffic lights. In fact, in some cities, the traffic lights are turned off (blinking yellow) at night so that cars can move straight ahead at intersections. Traffic lights become effective in regulating the traffic when the car density is high enough for northbound cars to interact with eastbound cars. Thus, in higher car densities, faulty traffic lights add randomness into the originally regulated traffic and lead to traffic jams. It is our aim to study the relation between the average car velocity and the concentration of faulty traffic lights and car density. In Sec. II, we define the CA model with faulty traffic lights. Results from numerical simulation are presented in Sec. III together with theoretical discussions. Results are summarized in Sec. IV.

II. MODEL

In our model, to simplify the definition and calculation of physical quantities, northbound cars will attempt a move on the first half of a time step, while eastbound cars will attempt on the second half. In this way, every car will have an opportunity to move in every time step. The velocity v_t at time t is then defined as the ratio between the number of cars moved at time t and the total number of cars. The traffic light on a site is assumed to regulate traffic entering that site. For example, on the first half of a time step, only northbound cars on the south may enter a site regulated by a traffic light. Let c be the fraction of faulty traffic lights in the lattice. We model the effects of faulty traffic lights as follows. For an empty site with a faulty traffic light, both northbound cars to the south and eastbound cars to the west of the site may attempt to enter, regardless of whether it is the first or second half of a time step. So, a northbound car to the south of a site with a faulty light will be able to move forward, even if it is the second half of a time step, provided that no eastbound car is trying to enter the same site simultaneously. The same goes for an eastbound car to the west of a site with a faulty light. In the case of two cars simultaneously attempting to enter, one of them will be chosen randomly to enter the site, and the other one will not move at that

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time step. With the introduction of faulty traffic lights, it becomes possible for a car to move twice in the same time step, resulting in a velocity larger than unity.

III. RESULTS

In our simulations, cars are placed in a 128×128 lattice with car density p. Periodic boundary conditions are applied. The number of northbound cars and the number of eastbound cars are assumed to be equal and are conserved under the boundary conditions. In the lattice, a fraction c of the sites are chosen to have faulty traffic lights. They are randomly distributed in the lattice. Each simulation is allowed to run until the velocity has reached a terminal value v, or until 5000 time steps have passed. In the latter case, the value of the terminal velocity is calculated by averaging over the last 128 time steps. We found that at most values of car densities, the average terminal velocity is reached well before 5000 time steps. At car densities near the transition between a high velocity phase (moving phase) and a vanishing velocity phase (jamming phase), more time steps are needed. However, the range of p separating the two phases is quite narrow. For a qualitative analysis of the effects of faulty traffic lights, we are merely looking at the change from a moving phase to a jamming phase instead of a precise determination of the critical car density. Thus, 5000 time steps suffice for our purpose. In cases where the velocity is still slightly fluctuating after 5000 time steps, averaging over 128 time steps gives a good representative value for the velocity. For every value of p and c, ten configurations with different initial conditions are averaged over.

In the case of no faulty traffic lights, our model reduces back to the BML model. The results of the BML model show two distinct phases separated by a critical car density p_c. For car densities below critical value ($p<p_c$), the system reaches a moving phase in the long time limit, with a nonzero terminal velocity of magnitude close to unity ($v\sim1$). For car densities above the critical value ($p>p_c$), the system reaches a jamming phase quickly, resulting in vanishing terminal velocity ($v=0$). Our model shows similar phase transitions from a moving phase to a jamming phase for all values of c. Figure 1 shows the average terminal velocity v as a function of p for different values of c. The range of c covers the Biham model ($c=0$) and the other extreme case of $c=1$ in which the lattice is full of faulty traffic lights. Every curve exhibits the reasonably sharp phase transition between the moving phase and jamming phase, as in the BML model. The velocity v at low car densities ($p\sim0$) *increases* as c increases, and reaches the value of $v=2$ at $c=1$. On the other hand, the value of the critical car density p_c *decreases* as c increases, and approaches $p_c\sim0.1$ at $c=1$.

For $p\sim0$, the chance of interactions between cars is low. A car can move forward twice when a site with a faulty traffic light is encountered. As a result, the average terminal velocity increases with the concentration of faulty lights. The values of v_0 of the terminal velocity at $p\sim0$ can be derived by neglecting interactions between cars. When a car travels across the whole lattice, it travels, on the average, over cN sites with faulty traffic lights.

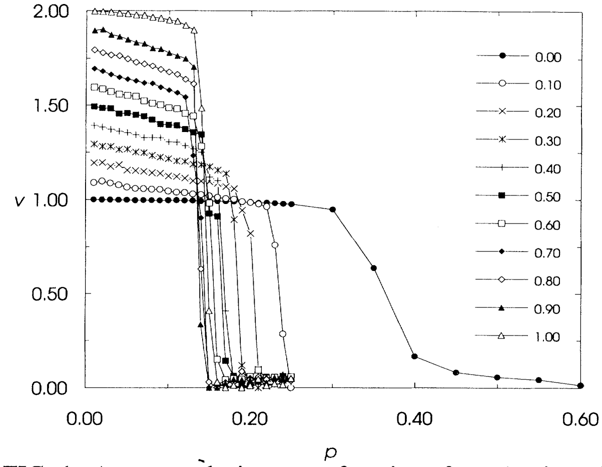

FIG. 1. Average velocity v as a function of car density p for different concentrations c of sites with faulty lights. Different curves refer to $c=0, 0.1, 0.2, \ldots, 1$.

On the sites with faulty traffic lights, it is able to move two sites in one time step, thus having a speed of 2 on these sites, and a speed of unity on other sites. Therefore, the average velocity is $[(1-c)N+2cN]/N=1+c$. Figure 2 shows the values of v_0 obtained by simulation, and the results agree with our theoretical prediction.

The decrease of the value of p_c as c increases can be understood qualitatively as follows. At $c=0$, for low car densities ($p<0.3$), the system evolves to a moving phase, and the cars form a moving pattern. For $c>0$, however, the presence of faulty traffic lights speeds up the cars occasionally, resulting in the jamming between cars and the disruption of the moving pattern. Thus, it is harder to form a moving pattern with the addition of faulty traffic lights, and the result is a lower p_c. For very high car densities, the traffic lights, either normal or faulty, become ineffective and the cars are jammed. Therefore, we see that in the intermediate range of car densities, the addition of faulty traffic lights leads to slower traffic in general and sometimes results in complete traffic jams. For example, for a car density of 0.2, the terminal velocity is unity for the case of no faulty traffic lights. However, for

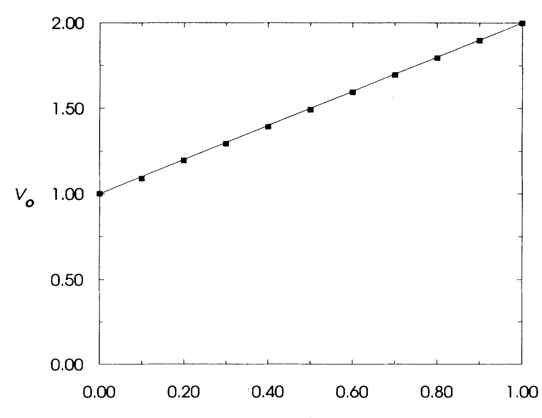

FIG. 2. Average velocity v_0 at vanishing p as a function of concentration c of sites with faulty lights. Square dots are the simulation results, and the solid line is the theoretical prediction of $v_0=1+c$.

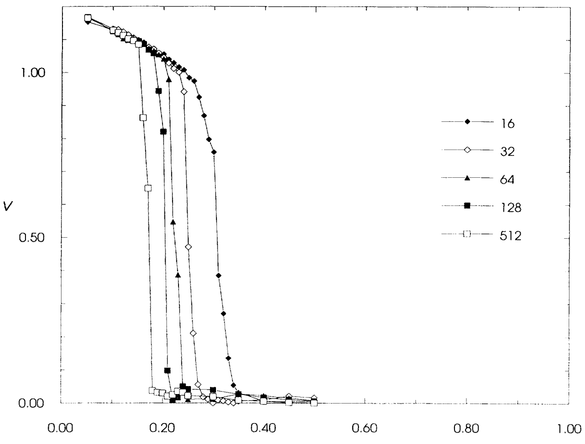

FIG. 3. Average velocity v as a function of car density p for five different lattice sizes at a fixed concentration $c = 0.2$ of faulty traffic lights. Different symbols refer to 16×16, 32×32, 64×64, 128×128, and 512×512 lattices, respectively.

a concentration of 20% of faulty lights, the average velocity drops to about 0.8, and for higher concentrations of faulty lights, the long time limit is a complete jamming phase.

For critical phenomena, it is expected that the size of the system plays an important role. It is, therefore, interesting to study the effects of lattice sizes on the transition between moving and jamming phases. Following the work of BML, we carried out simulations at different lattice sizes in the presence of faulty traffic lights. Figure 3 shows the results at a fixed concentration of faulty traffic lights of $c = 0.2$ for five different lattice sizes up to 512×512 lattices. Results are similar to that of the ordered case [1]. The transition becomes sharper as the lattice size increases and the critical transition car density shifts downward.

IV. SUMMARY

In summary, using a CA model, we have studied the effects of faulty traffic lights in a 2D traffic flow problem. Introducing faulty traffic lights increases the average velocity at low car densities, but also decreases the critical density at which the jamming phase occurs. The velocity at vanishing car density for different concentrations of sites with faulty traffic lights can be explained by simply neglecting the interactions between cars.

In reality, at places with faulty traffic lights and traffic jams, drivers tend to try to make a turn around the street and bypass the busy spot. Thus, in more realistic models, one should introduce the possibility of turning a northbound car into an eastbound car. Another complication is that it is not always true that the concentration of cars going to the north is the same as that going to the east. This may due to the planning and distribution of residential and business districts in a city. Studies of these two cases by slightly modifying the present model are now underway and results will be reported in future publications.

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